

Scientific Visualization – Homework 1

Submission: May, 7th, 2020, 10:15 am, via email

1. Exercise

(6 points)

Let $\mathcal{P} := \{P_0, \dots, P_4\} \subset \mathbb{R}^2$ be a set of control points given by

$$P_0 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, P_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, P_2 = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, P_3 = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, P_4 = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \in \mathbb{R}^2.$$

- 1.) Determine all resulting control points using the *de Casteljau algorithm* for $t = \frac{1}{2}$. Determine $b(t)$ explicitly. Represent the given control points, the resulting control points, and $b(t)$ graphically.
- 2.) Let $\{m_0, \dots, m_4\}$ be a set of tangent directions belonging to the points in \mathcal{P} given by

$$m_0 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, m_1 = \frac{1}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix}, m_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, m_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, m_4 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

Determine a C^1 piecewise defined cubic polynomial passing through the given points with the given tangent directions. Illustrate your results.

2. Exercise

(5 points)

Show the following properties of the *Bernstein polynomials* ($n \in \mathbb{N}_0$, $i \in \{0, \dots, n\}$):

- 1.) $B_i^n(t)$ has exactly one maximum in $[0, 1]$,
- 2.) $B_i^n(t) = \frac{i+1}{n+1} B_{i+1}^{n+1}(t) + \frac{n+1-i}{n+1} B_i^{n+1}(t)$,
- 3.) $B_i^m(t) B_j^n(t) = \frac{\binom{m}{i} \binom{n}{j}}{\binom{m+n}{i+j}} B_{i+j}^{m+n}(t)$, $m \in \mathbb{N}_0$, $j \in \{0, \dots, n\}$.

Please, turn over.

3. Exercise

(5 points)

Get access to some interpreter for *postscript*^I Based on the code presented in the lecture, use your initials to write a routine which represents them in manner as shown in Fig. (1).

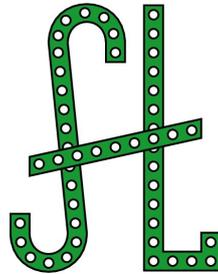


Abbildung 1: An illustrating example.

Send in an executable ps-file. Please name it as follows: 01_YourName.ps.

Total: 16

^ITherefore, you may visit <https://www.ghostscript.com/faq.html>.