

Differential Geometry II – Homework 08

Submission: June 27th, 2018, 12:15 am

1. Exercise (9 points)

Let M and \tilde{M} be two geodesically complete, connected Riemannian manifolds and let $\pi : \tilde{M} \rightarrow M$ be a *local isometry*, i.e. for each point $p \in \tilde{M}$, there exists an open neighborhood $U \subseteq \tilde{M}$ of p such that $\pi|_U$ is an isometry.

- 1.) Show that π fulfils the *lifting property for geodesics*: for every geodesic $\gamma : [0, 1] \rightarrow M$ in M and each point $p \in \tilde{M}$ with $\pi(p) = \gamma(0)$ there exists a unique geodesic $\tilde{\gamma} : [0, 1] \rightarrow \tilde{M}$ such that $\pi(\tilde{\gamma}) = \gamma$ and $\tilde{\gamma}(0) = p$.
- 2.) Show that π is surjective.
- 3.) Conclude that π is a smooth covering map.

2. Exercise (7 points)

Consider the n -dimensional open unit ball D equipped with the metric $g_{-1}|_p := \frac{4}{(1-|p|^2)^2} \delta_{ij}$. A *horosphere* in this model is (Euclidean) sphere placed in the interior of D such that it touches ∂D in a single point, for instance the image S under the map

$$f(u, v) := \frac{1}{2}(\cos(u) \sin(v), 1 - \cos(v), -\sin(u) \sin(v))$$

for the disk model in dimension $n = 3$.

- 1.) Sketch the model containing S .
- 2.) Determine $g_{ij} := g_{-1}(\partial_i f, \partial_j f)$ and the Christoffel symbols.
- 3.) Show $\partial_u \Gamma_{12}^2 - \partial_v \Gamma_{11}^2 + \Gamma_{12}^1 \Gamma_{11}^2 + \Gamma_{12}^2 \Gamma_{12}^2 - \Gamma_{11}^2 \Gamma_{22}^2 - \Gamma_{11}^1 \Gamma_{12}^2 = 0$.
- 4.) Show that S is flat.

Total: 16