Freie Universität Berlin SoSe 18
Institut für Mathematik
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Differential Geometry II – Homework 06

Submission: June 13, 2018, 12:15 am

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1. Exercise (5 points)
Let (M, g) be a Riemannian manifold and let $\{E_1, \ldots, E_{n-1}, X\}$ be an orthonormal basis of T_pM at some point p in M. Denote with $(\mathbb{T}_p)_i$ the plane in T_pM spanned by E_i and X. Show that

$$\mathrm{Ric}(X, X) = \sum_{i=1}^{n-1} K_{(\mathbb{T}_p)_i}$$

where $K_{(\mathbb{T}_p)_i}$ denotes the sectional curvature.

2. Exercise (6 points)
Consider the manifold $\mathbb{R}^n$ equipped with the hyperbolic metric

$$g_{-1}|_p := \frac{4}{(1-|p|^2)^2}\delta_{ij}.$$

Determine

1.) the curvature tensor R,

2.) the Ricci curvature Ric,

3.) the sectional curvature K,

4.) and the scalar curvature S

of $(\mathbb{R}^n, g_{-1})$.

3. Exercise (5 points)
Consider the upper half plane $\mathbb{H} := \{(x, y) \in \mathbb{R}^2 | y > 0\{$ equipped with the metric

$$g_p := \frac{1}{y^q}\delta_{ij}$$

where $q \in \mathbb{R}_+$ and $q \neq 2$. Show that $(\mathbb{H}, g_q)$ is not geodesically complete.

Total: 16