

## Differential Geometry II – Homework 01

Submission: April 30, 2018, 12:15 am

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### 1. Exercise

(16 points)

Consider the following smooth surface patch

$$\zeta : (1, 2) \times [0, 2\pi] \rightarrow \mathbb{R}^3, \\ (r, \varphi) \mapsto \begin{pmatrix} r \cos(\varphi) \\ r \sin(\varphi) \\ r \end{pmatrix},$$

and the following two open sets in  $\mathbb{R}^2$

$$\mathcal{O}_1 = \left\{ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \mid 1 < |x| < 2, x_1, x_2 > 0 \right\}, \\ \mathcal{O}_2 = \{ x \in \mathbb{R}^2 \mid 0 < x_1, x_2 < 1 \}.$$

- 1.) Sketch  $\mathcal{O}_1$ ,  $\mathcal{O}_2$ , and  $\zeta((1, 2) \times [0, 2\pi])$ .
- 2.) Determine charts  $\varphi_1 : \mathcal{O}_1 \rightarrow \mathbb{R}^3$  and  $\varphi_2 : \mathcal{O}_2 \rightarrow \mathbb{R}^3$  such that they cover  $\zeta((1, 2) \times [0, 2\pi])$ .
- 3.) Justify your solution of 2.) via showing that you constructed an atlas for  $\zeta((1, 2) \times [0, 2\pi])$ .
- 4.) Determine the corresponding *coordinate functions* in both charts.
- 5.) Determine a basis for the *tangent space*. Justify your solution.

Total: 16