Freie Universität Berlin WiSe 18/19
Institut für Mathematik
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Differential Geometry III – Homework 08

Submission: January 9, 2019, 12:15 am

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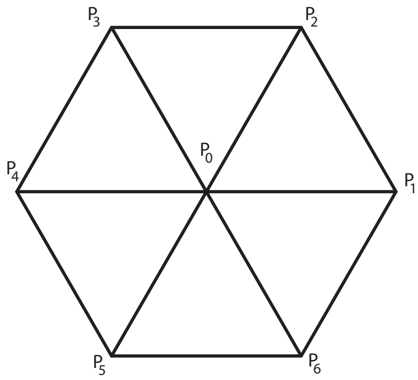
1. Exercise (4 points)

Consider the following triangulation with vertices

$$P_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, P_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, P_2 = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}, P_3 = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}, P_4 = \begin{pmatrix} -1 \\ 0 \end{pmatrix},$$
$$P_5 = \begin{pmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} P_6 = \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix}$$

and let $u \in S_h$ be a continuous, piecewise (affine) linear function defined by $u_0 = 1$, $u_1 = 1$, $u_2 = 1$, $u_3 = \frac{1}{2}$, $u_4 = \frac{1}{3}$, $u_5 = 2$ and $u_6 = -3$, where $u_i \coloneqq u(P_i)$.



1.) Determine the gradient field ∇u and sketch it.

2.) For the triangle $T_0 = [P_0, P_1, P_2]$, let $w \in S_h$ be a function with $w_1 = 1$, $w_2 = -1$ and gradient $\nabla w|_{T_0} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$. Determine w_0.

2. Exercise (4 points)

Let M_h be a simply connected simplicial surface and let v be a vector field on M_h. Further let $p \in M_h^{(0)}$ and $c_i \in M_h^{(1)}$. Show the following statements from the lecture:

1.) $\operatorname{div}(v)|_p = \frac{1}{2} \sum_{c_i \in M_h^{(1)} : p \in c_i} \operatorname{div}^*(v)|_{c_i}$,

2.) $\operatorname{curl}(Jv) = \operatorname{div}(v)$,

3.) $\operatorname{curl}^*(Jv) = \operatorname{div}^*(v)$,

where J denotes the rotation by $\frac{\pi}{2}$ in the oriented tangent space.

Total: 8