Freie Universität Berlin WiSe 18/19
Institut für Mathematik
Prof. Dr. K. Polthier, H.-S. Lipschütz Status: October 24, 2018

Differential Geometry III – Homework 01

Submission: October 31, 2018, 12:15 am

---

1. Exercise (4 points)

Let X be a topological space and $x_0 \in X$ a point in X. The set of closed paths starting at x_0 will be denoted by

$$\Gamma_{x_0} := \{\gamma : [0,1] \to X \mid \gamma(0) = \gamma(1) = x_0\}.$$

Let e, $\gamma_1, \gamma_1, \gamma_1', \gamma_2, \gamma_2' \in \Gamma_{x_0}$ where $e = x_0$ denotes the *constant path*. Prove the following statements:

1.) Homotopy equivalence defines an equivalence relation.

2.) $e \circ \gamma \simeq \gamma \circ e \simeq \gamma$.

3.) Let $\gamma^{-1}(s) := \gamma(1-s)$ denote the *inverse path*. Then $\gamma^{-1} \circ \gamma \simeq \gamma \circ \gamma^{-1} \simeq e$.

4.) If $\gamma_1 \simeq \gamma_1'$ and $\gamma_2 \simeq \gamma_2'$ then $\gamma_1 \circ \gamma_2 \simeq \gamma_1' \circ \gamma_2'$.

5.) The composition of paths $[\gamma_1] \circ [\gamma_2] = [\gamma_1 \circ \gamma_2]$ is well-defined.

2. Exercise (4 points)

Show that the following representations S_1 and S_2 of $SU(2)$ are equal indeed where

$$S_1 := \{A \in \mathrm{Gl}(2, \mathbb{C}) \mid A^H A = I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \det(A) = 1\}$$

and

$$S_2 := \{\begin{pmatrix} a & b \\ -\overline{b} & \overline{a} \end{pmatrix} \mid |a|^2 + |b|^2 = 1,\ a, b \in \mathbb{C}\}.$$

Total: 8