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WiSe 17/18

Status: February 7, 2018

Differential Geometry I – Homework 13

Submission: February, 14, 2018, 12:15 am

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1. Exercise (8 points)

Let $S := \{A \in \mathbb{R}^{3,3} | A = -A^T\}$ denote the set of skew-symmetric matrices and let

$$\begin{aligned} \text{Cay} : S &\to \text{SO}(3) \\ A &\mapsto \text{Cay}(A) = (I + A)(I - A)^{-1} \end{aligned}$$

be the *Cayley map*.

Show the following statements (for $A \in S$):

1.) for $\mu \in \mathbb{R}_+$, A and $(\mu I \pm A)^{-1}$ commute,

2.) $Q = (I + A)(I - A)^{-1}$ is an orthogonal matrix,

3.) let P be an orthogonal matrix which has no eigenvalue equal to -1, then $(I - P)(I + P)^{-1} \in S$,

4.) determine an expression for $\text{Cay}(A)^{-1}$. Justify your solution.

5.) Is Cay a surjective map? Justify your answer.

2. Exercise (8 points)

Let M be a differentiable manifold. A tangent vector at $p \in M$ is an equivalence class of differentiable curves ($\epsilon > 0$) – the so called *germs* —

$$[c] := \{c : (-\epsilon, \epsilon) \to M \mid c(0) = p\}/\sim$$

where $c \sim \tilde{c}$ if and only if $(\varphi \circ c)'(0) = (\varphi \circ \tilde{c})'(0)$ for every chart φ.

1.) Show that $\sim$ is indeed an equivalence relation.

2.) Show that the set of germs forms a real vector space.

Total: 16