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Differential Geometry I – Homework 11

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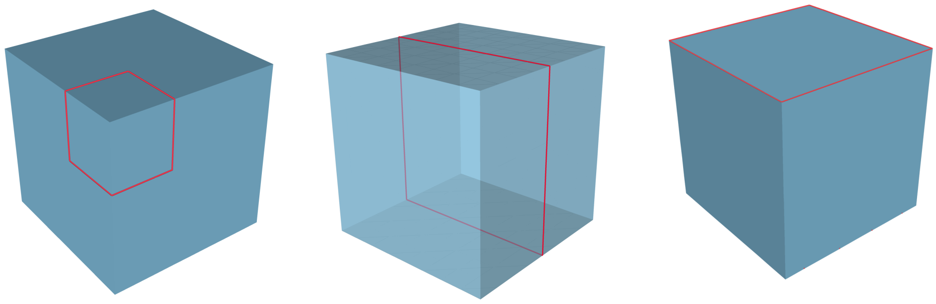
1. Exercise (6 points)

Let $\gamma_i : [0,1] \to M_h$ be the closed curves on the cube's surface M_h shown below. Let v be an initial tangent vector at $\gamma_i(0)$, $i \in \{1,2,3\}$.



1.) In all three cases, compute the angle defect $\beta_N^i(1) - \beta_N^i(0)$ for the parallel transport of v along γ_i as well as the integrals $\int_0^1 \kappa_{g,i}(t)\mathrm{d}t$.

2.) Verify the discrete Gauß-Bonnet-formula for the region bounded by γ_i in all three cases.[1]

2. Exercise (4 points)

Let X be vector field along a geodesic γ.
Show: X is parallel along γ if and only if X has got constant length and X encloses a constant angle with $\dot{\gamma}$.

[1] Choose your preferred orientation.

3. Exercise (4 points)

Use the Gauß-Bonnet-theorem to prove the following claims.

1.) The sum of angles of a Euclidean triangle is equal to π.

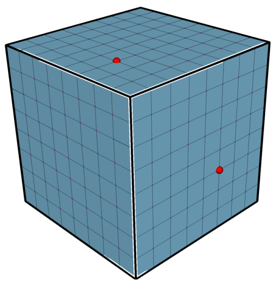
2.) Let c be a simple[2], closed planar curve. Then $\int_c \kappa_g = 2\pi$.

3.) On a simply connected surface S with negative Gaussian curvature two geodesics emanating from a point $p \in S$ do not meet in a second point $q \in S$.

4.) A surface homeomorphic to a torus always has a point of vanishing Gaussian curvature.

4. Exercise (2 points)

Find and sketch two discrete geodesics connecting the two highlighted points–one of them being a shortest, one not being a shortest geodesic. Justify your solution.



Total: 16

[2]I.e., there are no self intersections.