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Differential Geometry I – Homework 09

Submission: January 15, 2018, 12:15 am

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1. Exercise (12 points)

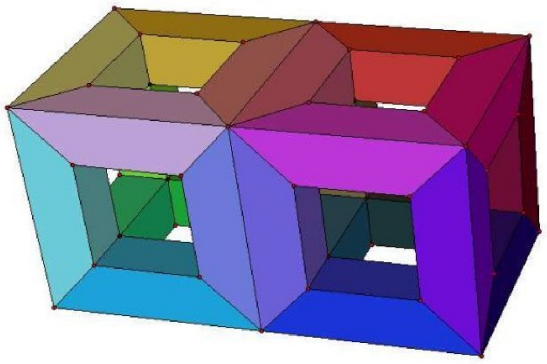
1.) Determine the genus of a simplicial surface with 20 triangles and 12 vertices.

2.) Sketch[1] two nonisomorphic examples of a closed simplicial surface with 20 triangles and 12 vertices which are both not the icosahedron. Why are your examples nonisomorphic?

3.) How many edges does a simplicial double torus with 1200 vertices have got?

4.) a) Let Q be a quadrangulated surfaces (such as the example shown in the next part of the exercise). Show that the Euler formula $\chi(Q) = v - e + f$ still holds.

b) Consider the following quadrangulated surface T:



Determine the number of vertices v, the number of edges e, the number of faces f and the Euler characteristic $\chi(T)$. Further, determine its genus $g(T)$. What topological surface is given?

5.) Give three examples for simplicial complexes having the same Euler characteristic but which are not simplicially isomorphic[2]. Justify your choice.

[1] Think of an appropriate representation.
[2] I.e., they are not isomorphic as simplicial complexes.

2. Exercise (4 points)

Let p be an inner point of a simplicial regular[3] surface S.

1.) Determine the number of triangles incident to p such that the discrete Gauss curvature K in p is equal to $\frac{2\pi}{3}$, 0, or $-\frac{2\pi}{3}$ resp. Illustrate your results.

2.) Determine the number of triangles incident to p such that $K(p) = 42\pi$ resp. $K(p) = -42\pi$.

3.) Determine the discrete Gauss curvature for the surface depicted in Exercise 1, 4.), b).

Total: 16

[3]All edges have the same length, and therefore, all angles are equal, too.