

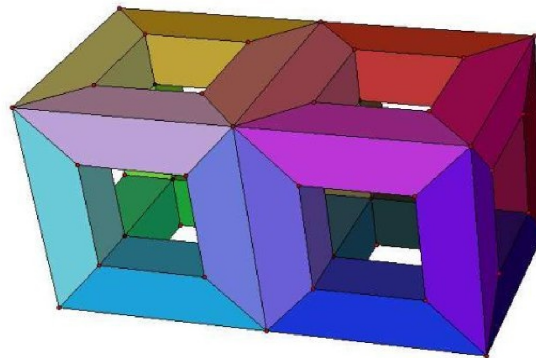
Differential Geometry I – Homework 09

Submission: January 15, 2018, 12:15 am

1. Exercise

(12 points)

- 1.) Determine the genus of a simplicial surface with 20 triangles and 12 vertices.
- 2.) Sketch¹ two nonisomorphic examples of a closed simplicial surface with 20 triangles and 12 vertices which are both not the icosahedron. Why are your examples nonisomorphic?
- 3.) How many edges does a simplicial double torus with 1200 vertices have got?
- 4.) a) Let Q be a quadrangulated surfaces (such as the example shown in the next part of the exercise). Show that the Euler formula $\chi(Q) = v - e + f$ still holds.
b) Consider the following quadrangulated surface T :



Determine the number of vertices v , the number of edges e , the number of faces f and the Euler characteristic $\chi(T)$. Further, determine its genus $g(T)$. What topological surface is given?

- 5.) Give three examples for simplicial complexes having the same Euler characteristic² but which are not simplicially isomorphic². Justify your choice.

¹Think of an appropriate representation.

²I.e., they are not isomorphic as simplicial complexes.

2. Exercise

(4 points)

Let p be an inner point of a simplicial regular³ surface S .

- 1.) Determine the number of triangles incident to p such that the discrete Gauss curvature K in p is equal to $\frac{2\pi}{3}$, 0 , or $-\frac{2\pi}{3}$ resp. Illustrate your results.
- 2.) Determine the number of triangles incident to p such that $K(p) = 42\pi$ resp. $K(p) = -42\pi$.
- 3.) Determine the discrete Gauss curvature for the surface depicted in Exercise 1, 4.), b).

Total: 16

³All edges have the same length, and therefore, all angles are equal, too.