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Differential Geometry I – Homework 06

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1. Exercise (6 points)

Let

$$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^3,\ (u,v) \mapsto (u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2).$$

Then f parametrizes the *Enneper surface*[1].

1.) Determine the principal curvatures and the mean curvature of f.

2.) Show: the coordinate lines are lines of curvature.

3.) Construct a parametrization by polar coordinates and show that rotations in the parameter domain are intrinsic isometries.

2. Exercise (5 points)

Let

$$f : \mathbb{R} \times [0, 2\pi) \to \mathbb{R}^3,\ (t, \varphi) \mapsto (r(t)\cos(\varphi), r(t)\sin(\varphi), h(t))$$

be a surface of revolution. Show that all meridians (φ constant) are geodesics, and that the circle of latitude (t constant) through a point $(r(t_0), 0, h(t_0))$ of the profile curve is a geodesic if and only if $r'(t_0) = 0$ holds.

3. Exercise (5 points)

Let $f \circ \gamma$ be a curve parametrized by arc length on a surface $f : \Omega \to \mathbb{R}^3$. Prove the following statements:

1.) γ'' is parallel to the normal of the surface if and only if γ is a geodesic.

2.) The curve γ is a straight line if and only if γ is an asymptotic line and a geodesic.

3.) If γ is a line of curvature and a geodesic then γ is a planar curve. Does the converse hold?

Total: 16

[1] It is a well known minimal surface.