Freie Universität Berlin WiSe 17/18
Institut für Mathematik
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Differential Geometry I – Homework 04

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1. Exercise (7 points)
For $t > 0$ consider the *tractrix*

$$(r, h)(t) := (\tfrac{1}{\cosh(t)}, t - \tanh(t))$$

and the corresponding surface of revolution[1]

$$f : \mathbb{R} \times [0, 2\pi),\ (t, \varphi) \mapsto (r(t)\cos(\varphi), r(t)\sin(\varphi), h(t)).$$

1.) Sketch the tractrix.
2.) Determine both principal curvatures κ_1 and κ_2.
3.) Show that its Gaussian curvature is constant.
4.) Determine its surface area.

2. Exercise (4 points)
1.) Show that a *surface of revolution* can always be parametrized such that $E = 1$, $F = 0$, and $G = g(t)$ where g is some function depending on t only.
2.) Show that a surface of revolution locally admits a *conformal parametrization*, i.e. $E = G$, and $F = 0$.

3. Exercise (5 points)
Let $f : \Omega \to \mathbb{R}^3$ be a parametrized surface with metric g, secind fundamental form b and shape operator S. in each $p \in \Omega$, the third fundamental form is a symmetric bilinear form h given by

$$h(v, w) := g(Sv, Sw) \text{ for all } v, w \in T_p\Omega.$$

Show the following equality

$$h(v, w) - 2Hb(v, w) + Kg(v, w) = 0,$$

where K denotes the Gaussian curvature and H denotes the mean curvature of $f(\Omega)$.

Total: 16

[1] It is called *tractroid* or *pseudosphere*.