**Freie Universität Berlin** WiSe 17/18
**Institut für Mathematik**
Prof. Dr. K. Polthier, H.-S. Lipschütz Status: November 14, 2017

# Differential Geometry I – Homework 04

Submission: November 20, 2017, 12:15 am

---

**1. Exercise** (7 points)

For $t > 0$ consider the *tractrix*

$$(r, h)(t) := (\tfrac{1}{\cosh(t)}, t - \tanh(t))$$

and the corresponding surface of revolution[1]

$$f : \mathbb{R} \times [0, 2\pi),\ (t, \varphi) \mapsto (r(t)\cos(\varphi), r(t)\sin(\varphi), h(t)).$$

1.) Sketch the tractrix.
2.) Determine both principal curvatures $\kappa_1$ and $\kappa_2$.
3.) Show that its Gaussian curvature is constant.
4.) Determine its surface area.

**2. Exercise** (4 points)

1.) Show that a *surface of revolution* can always be parametrized such that $E = 1$, $F = 0$, and $G = g(t)$ where $g$ is some function depending on $t$ only.
2.) Show that a surface of revolution locally admits a *conformal parametrization*, i.e. $E = G$, and $F = 0$.

**3. Exercise** (5 points)

Let $f : \Omega \to \mathbb{R}^3$ be a parametrized surface with metric $g$, secind fundamental form $b$ and shape operator $S$. in each $p \in \Omega$, the third fundamental form is a symmetric bilinear form $h$ given by

$$h(v, w) := g(Sv, Sw) \text{ for all } v, w \in T_p\Omega.$$

Show the following equality

$$h(v, w) - Hb(v, w) + Kg(v, w) = 0,$$

where $K$ denotes the Gaussian curvature and $H$ denotes the mean curvature of $f(\Omega)$.

Total: 16

[1] It is called *tractroid* or *pseudosphere*.