Freie Universität Berlin WiSe 17/18
Fachbereich Mathematik und Informatik
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Differential Geometry I – Homework 02

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1. Exercise (8 points)

The following map

$$\eta : [0, 2\pi] \to \mathbb{R}^3,$$
$$t \mapsto \begin{pmatrix} r\cos(t) \\ r\sin(t) \\ ht \end{pmatrix}$$

parametrizes a so called *helix* with radius $r > 0$ and slope $h \in \mathbb{R}$.

1.) Determine $\mathrm{L}(\eta|_{[0,2\pi]})$.
2.) Find a parametrization by arc length of η.
3.) Determine all r and h (depending on r) such that η is parametrized by arc length already. Sketch your results.
4.) Determine the curvature and torsion of η.
5.) (4 additional points) Use some software (Mathematica, Matlab, JavaView, etc.) to depict and to compare the tubes generated by the Frenet frame and by a parallel frame of η.

2. Exercise (4 points)

Let $\gamma : \mathbb{R} \to \mathbb{R}^2$ be a regular curve. Show: if all tangents of γ intersect in one point $P \in \mathbb{R}^2$ then γ is a straight line.

3. Exercise (4 points)

Let $\gamma : [a, b] \to \mathbb{R}^n$ be a regular curve. Show: for all $\epsilon > 0$, there exists $\delta > 0$ such that for any subdivision $a \le t_0 < t_1 < \ldots < t_n \le b$ with resolution $\max_i |t_{i+1} - t_i| < \delta$ the maximum distance between curve and polygon $P = (\gamma(t_0), \ldots, \gamma(t_n))$ is smaller than ϵ:

$$\max_{t \in [a,b]} |\gamma(t) - P(t)| < \epsilon.$$

Gesamtpunktzahl: 16