Prof. Dr. Konrad Polthier
Dipl.-Math. Konstantin Poelke

Differential Geometry II
Summer Semester 2015
Freie Universität Berlin

Exercise Sheet 1

Online: 15.04.2015

Due: 22.04.2015, 4:00pm in the Tutorials

Exercise 1.1 (Level Sets, 4 Points)**.** Let $F : \mathbb{R}^n \to \mathbb{R}^{n-k}$ be a smooth submersion (i.e. the Jacobian DF has rank $n-k$) and let $S := \{x \in \mathbb{R}^n : F(x) = 0\}$. Prove that S is a manifold.

Hint: Implicit function theorem.

Exercise 1.2 (Charts, 4 Points)**.** How many charts does an atlas for

1. ...the real projective plane $\mathbb{R}P^2 := \mathbb{S}^2/\sim$ where $x \sim y \Leftrightarrow y = -x$;
2. ...the Torus $T^2 := \mathbb{R}^2/\mathbb{Z}^2$

contain at least? Justify your answer and find such an atlas.

Exercise 1.3 (Submanifolds are Manifolds, 2 Points)**.** Let $S \subset N$ be a regular submanifold [1] of dimension k and let $\mathcal{A} = \{(U, \varphi)\}$ be an atlas for N. Prove that $\{(U \cap S, \varphi_S)\}$ forms an atlas for S.

[1] A subset $S \subset N$ of a manifold N of dimension n is a *regular (or embedded) submanifold of dimension* k if for every $p \in S$ there is a chart $(U, \varphi = (\varphi_1, \ldots, \varphi_n))$ for N with $p \in U$ such that $U \cap S$ is the set defined by the vanishing of $n-k$ coordinate functions φ_i. W.l.o.g. (i.e. after renumbering) we may assume that these $n-k$ functions are $\varphi_{k+1}, \ldots, \varphi_n$, i.e. $U \cap S = \varphi^{-1}(\varphi(U) \cap \mathbb{R}^k \times \{0\}\times, \ldots, \times\{0\})$. The chart $\varphi_S = (\varphi_1, \ldots, \varphi_k) : U \cap S \to \mathbb{R}^k$ is called an *adapted chart relative to* S. The number $(n-k)$ is the *codimension* of S in N.