

Exercise Sheet 9

Online: 10.12.2014

Due: 17.12.2014, 4:00pm

Four points for each exercise!

Exercise 9.1 (Gauss Map I). Let $f : U \rightarrow \mathbb{R}^3$ be a parametrized surface, $B \subset U$ a compact subset and let $N : U \rightarrow S^2$ denote the Gauss map of f . Assume that N is injective and DN is of full rank. Show that

$$\int_{f(B)} |K| = \text{Area}(N(B))$$

where $\text{Area}(N(B))$ denotes the area of the surface patch on S^2 swept out by the Gauss map on B .

Exercise 9.2 (Gauss Map II). Dropped!

Exercise 9.3 (Geodesics on a Torus). Describe the initial value problem for geodesics on a torus

$$f : (u, v) \mapsto (\cos u(R + r \cos v), \sin u(R + r \cos v), r \sin v).$$

Use the differential equations to verify that the following curves on the torus are geodesics:

1. $t \mapsto f(u_0, t)$ for a constant u_0 ;
2. $t \mapsto f(t, 0)$;
3. $t \mapsto f(t, \pi)$.