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Differential Geometry I
Winter Semester 2014/15
Freie Universität Berlin

Exercise Sheet 8

Online: 02.12.2014

Due: 10.12.2014, 4:00pm

Four points for each exercise!

Exercise 8.1 (Geodesics I)**.** On a surface of revolution

$$f(t,\varphi) = (r(t)\cos\varphi, r(t)\sin\varphi, h(t)),$$

show that all meridians (φ =constant) are geodesics, but that the circle of latitude (t = constant) through a point $(r(t_0), 0, h(t_0))$ of the profile curve is a geodesic if and only if $r'(t_0) = 0$ holds.

Exercise 8.2 (Geodesics II)**.** Let $\gamma = f \circ c$ be an arc-length parametrized curve that is contained in a surface $f : U \to \mathbb{R}^3$. Prove the following facts!

1. If γ'' is parallel to the normal of the surface, then γ is a geodesic.
2. The curve γ is a straight line if and only if it is both an asymptotic line and a geodesic.
3. If γ is a line of curvature and a geodesic, then γ is a planar curve. Does the converse hold?

Exercise 8.3 (Planar Geodesics)**.** Prove that if all geodesics of a connected surface are planar curves, then the surface is contained in a plane or a sphere.

Exercise 8.4 (Existence of Surfaces)**.** Is there a surface $f : U \to \mathbb{R}^3$ with the following first and second fundamental forms?

1. $(g_{ij})\,|_{(u,v)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $(h_{ij})\,|_{(u,v)} = \begin{pmatrix} 0 & 0 \\ 0 & u \end{pmatrix}$;
2. $(g_{ij})\,|_{(u,v)} = \begin{pmatrix} 1 & 0 \\ 0 & \cos^2 u \end{pmatrix}$ and $(h_{ij})\,|_{(u,v)} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2 u \end{pmatrix}$.