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Differential Geometry I
Winter Semester 2014/15
Freie Universität Berlin

Exercise Sheet 7

Online: 26.11.2014

Due: 03.12.2014, 4:00pm

Four points for each exercise!

Exercise 7.1 (Pseudosphere). Let $r(t) = a\mathrm{e}^t$ and $h(t) = \int_0^t \sqrt{1 - a^2\mathrm{e}^{2x}}dx$ for some constant $a \in \mathbb{R}$. The resulting surface of revolution is called a *pseudosphere*.

Determine the asymptotic lines and the curvature lines for the pseudosphere and show that the Gaussian curvature is -1 everywhere.

Visualize the asymptotic and curvature lines with the help of a computer (1 bonus point).

Exercise 7.2 (Asymptotic Lines on Minimal Surfaces). Consider a parametrized surface S with negative Gaussian curvature $K < 0$ everywhere. Prove the following statements:

1. At each point $q \in S$ there are two linearly independent *asymptotic directions*, i.e. two linearly independent tangent vectors $X, Y \in T_qS$ such that $I\!I(X, X) = I\!I(Y, Y) = 0$.
2. S is a minimal surface if and only if the asymptotic directions in each point on S are perpendicular to each other.

Exercise 7.3 (Christoffel Symbols). Let $f : U \to \mathbb{R}^3$ be a parametrized surface and assume that $g_{ij} = 0$ for $i \neq j$. Derive formulas for the Christoffel symbols Γ_{ij}^k of f. Use these formulas to compute the Christoffel symbols for the following parametrizations:

1. the sphere $f(u, v) = r(\cos u \cos v, \sin u \cos v, \sin v)$;
2. the sphere (stereographic projection) $f(u, v) = \frac{2}{u^2+v^2+4}(2u, 2v, u^2 + v^2)$;
3. the torus $f(u, v) = (\cos u(R + r\cos v), \sin u(R + r\cos v), r\sin v$.