Exercise Sheet 6

Online: 17.11.2014

Due: 26.11.2014, 4:00pm

Four points for each exercise!

Exercise 6.1 (Discrete Gauss Curvature)**.** Let M_h be a closed surface triangulated by a finite triangulation $\mathcal{T}$. For a vertex $p \in \mathcal{T}$ we define the discrete graph Gauss curvature K_h by

$$K_h(p) := 6 - \mathrm{val}(p),$$

where $\mathrm{val}(p)$ denotes the valence of the vertex p in the 1-*skeleton graph* of $\mathcal{T}$, i.e. the number of adjacent edges. Show that the following version of a discrete Gauss-Bonnet theorem holds:

$$\int_{M_h} K_h := \sum_{p \in \mathcal{T}} K_h(p) = 6\chi(M_h).$$

Exercise 6.2 (Two important observations)**.** Prove the following facts!

1. Let X_1 and X_2 denote the principal curvature directions on a surface at a given point and assume that the corresponding principal curvatures are not equal, i.e. $\kappa_1 \neq \kappa_2$. Then X_1 and X_2 are orthogonal.
2. There is no asymptotic line passing through an elliptic point on a surface (this will justify the well-posedness of the next exercise).

Exercise 6.3 (Asymptotic Lines)**.** Prove that the absolute value of the torsion τ at a point of an asymptotic line, whose curvature is nowhere zero, is given by

$$|\tau| = \sqrt{-K}$$

where K is the Gaussian curvature of the surface at the given point.

Exercise 6.4 (Weierstrass Parametrizations)**.** Consider the following pairs of complex functions $F, G : U \subset \mathbb{C} \to \mathbb{C}$:

1. $F(z) = 1$, $G(z) = z$ with $U = \mathbb{C}$;
2. $F(z) = 1$, $G(z) = \frac{1}{z}$ with $U = \mathbb{C} \setminus \{0\}$.

Use the Weierstrass representation to compute from F and G a parametrization of the corresponding minimal surfaces and plot the surfaces.