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Differential Geometry I
Winter Semester 2014/15
Freie Universität Berlin

Exercise Sheet 13

Online: 21.01.2015

Due: 28.01.2015, 4:00pm

Four points for each exercise! This is the last exercise sheet.

Exercise 13.1 (Points of positive Gauss curvature)**.** Let $S \subset \mathbb{R}^3$ be a closed, compact regular surface. Prove that there is a point $p_0 \in S$ with positive Gauss curvature $K(p_0) > 0$. Conclude that there is no closed, compact minimal surface in $\mathbb{R}^3$.

Exercise 13.2 (Geodesic Triangles in H^2)**.** Let T be a geodesic triangle in the hyperbolic plane with angles α, β and γ. Compute the area of T.

Exercise 13.3 (Discrete Parallel Transport)**.** Let $\gamma_i : [0,1] \to M_h$ be the closed curves (i.e. $\gamma_i(0) = \gamma_i(1)$) on the cube surface M_h shown in the figure below, $i = 1, \ldots, 4$, and let v be an initial tangent vector inside a face at $\gamma_i(0)$.

Compute for all cases the angle defect $\beta_i^N(1) - \beta_i^N(0)$ for the parallel transport of v along γ_i as well as the integrals $\int_0^1 \kappa_{g,i}(t)dt$ to verify the formula from the lecture (here $\kappa_{g,i}$ denotes the discrete geodesic curvature of the curve γ_i).

Exercise 13.4 (Discrete Gauss-Bonnet)**.** Verify the discrete Gauss-Bonnet formula for the region bounded by γ_i from Exercise 13.3 in all four cases (you may choose the "outer" or "inner" region in each case, depending on your preferred orientation).

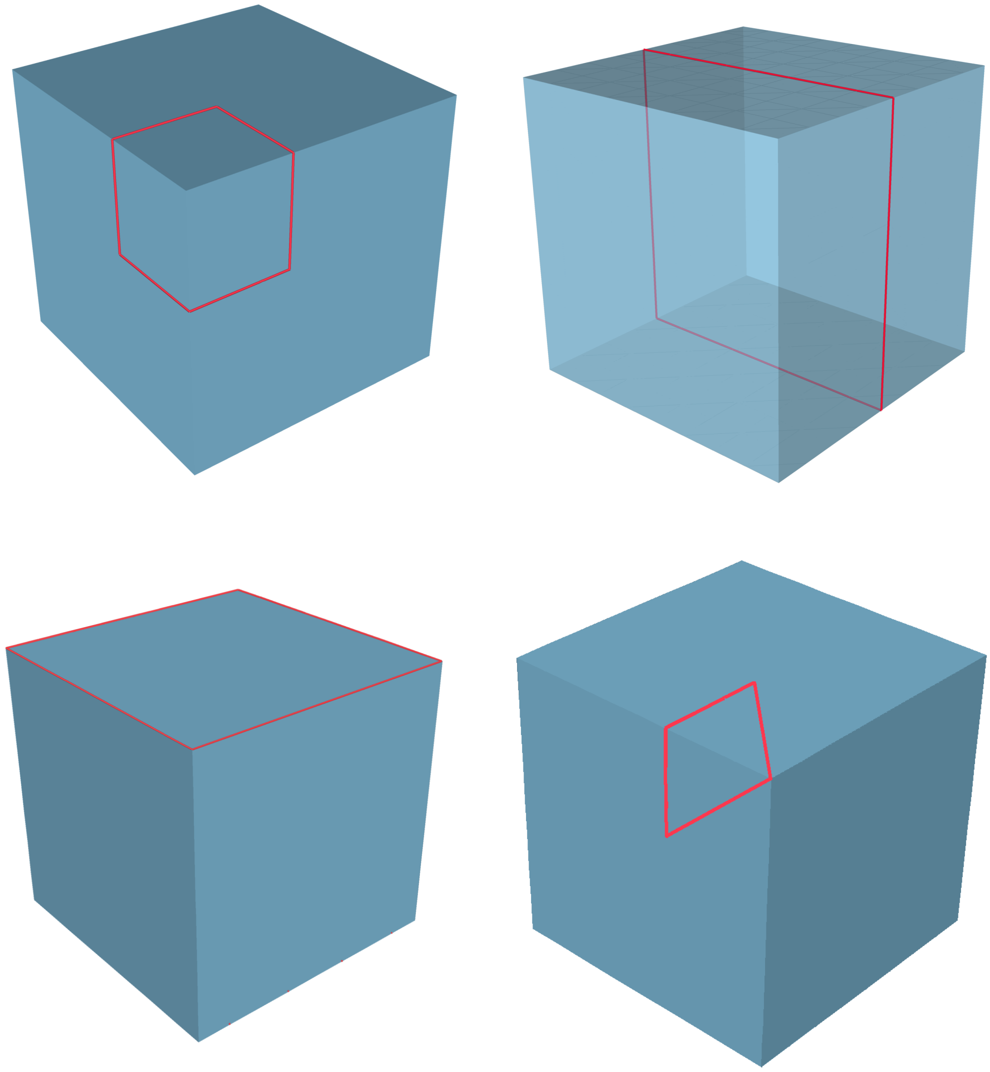

Figure 1: The curves γ_i on the cube.