F
Visualization

Konrad Polthier, John Sullivan, Günter M. Ziegler and Hans-Christian Hege

Summary. The emphasis on visualization as a separate discipline and application area is a unique selling point of the DFG Research Center MATHEON. Visualization techniques open an informative (and attractive) window into mathematical research, scientific simulations and industrial applications. The application area Visualization focuses on key problems in the fields of geometry processing, medical image processing and mathematical visualization. Geometric algorithms are key to many industrial technologies, including computer-aided design (CAD), image and geometry processing, computer graphics, numerical simulations and animations involving large-scale datasets. For all these technologies, a deep understanding of the underlying abstract mathematical concepts is essential. Notable has been the effort in recent years by the CAD industry to rethink the foundations of their work and put it on a firmer mathematical footing. Today, progress in the development and use of new mathematical concepts is often what characterizes state-of-the-art applications. Mathematical knowledge then becomes a key resource within industry to stay competitive.

Guiding research problems. The research in this application area is mainly driven by challenging mathematical problems in computer graphics, medicine, and industrial computer aided design, which require new mathematical concepts and geometric algorithms. We mention three example challenges: the reverse engineering process from digitizing a 3D model to a functional, virtual 3D model has been characterized as "one of the most challenging problems" in the CAD industry. Here new differential geometric concepts for polyhedral meshes are required in combination with new anisotropic meshing algorithms for surfaces and efficient adaptive numerical implementations. Another challenge addresses the resource requests of today's computer generated feature films. These often rely on gigantic virtual scenes, which often do not fit into main memory and cannot be rendered in a single pass. Feature-based mesh-reduction techniques and optimally efficient data compression algorithms are needed to reduce the redundancy of mesh representations by many orders of magnitude. The third challenge deals with the computer aided surgery planning. Currently, medical doctors have few objective criteria for the surgical reshaping procedures. Here computer simulations based on statistical 3D models are promising and have been successfully applied to surgery of craniosynostosis, that is, skull deformities in the growth process of infants due to premature ossification of cranial sutures.

Brief introductions of the three main research domains of this application area are followed by longer expositions discussing in more detail the central research problems and industrial applications.

Geometry processing. In computer graphics, industrial geometric design, scientific computing and digital geometry processing, curved surfaces in 3D space are usually represented by complicated triangle or polyhedral meshes. Processing and simulating such surface meshes is a key challenge, both theoretically from the mathematical perspective and from the perspective of industrial design (including CAD, architecture and computer graphics). Mathematically, we ask for the best way to discretize classical differential geometry. Practical questions include finding simple, robust and efficient data structures and algorithms for digital surfaces.

Most projects are engaged in different aspects of discrete differential geometry. On the theoretical front, classical concepts from smooth differential geometry – such as curvatures and differential operators – are translated into corresponding discrete notions. It remains difficult to transfer results from discrete differential geometry to the industrial environment where spline surfaces still dominate the CAD market. A special challenge is to develop efficient algorithms to optimize, process, and simulate discrete surfaces with millions of degrees of freedom.

A fundamental problem that appears in many applications is to parametrize such surface meshes, i.e., to map them to planar domains. For example, such parametrizations are used for texture mapping in computer graphics and to discretize partial differential equations. As in the case of smooth surfaces, it is desirable that the parametrization is adapted to the geometry of the surface. Smooth surfaces possess a number of adapted parametrizations that are well known from classical differential geometry: conformal parametrizations, parametrizations by curvature lines or asymptotic lines, etc. This raises very natural questions regarding discrete surfaces (meshes), like for example: "What does it mean for a discrete surface to be parametrized by curvature lines?"

One approach to the parametrization problem is to discretize the differential equations satisfied by a special parametrization in some standard way. However, the key challenge is to find proper discrete analogues of differential geometric notions and to develop the corresponding theory. A discrete curvature-line mesh should have the desired geometrical properties already at the coarse level and not only in some limit of refinement as it approaches a smooth curvature-line parametrized surface.

Image processing. Many medical imaging systems (such as CT and MRT) require novel techniques for image processing and segmentation. The problem of recognizing anatomical objects in such images is crucial for efficient diagnostics. We focus on image segmentation from two different points of view: on one side, we make use of a-priori knowledge about geometric and topological shape characteristics to target 2D segmentation and 3D geometry reconstruction. In a different project we treat methods and models for the analysis of diffusion weighted imaging (DWI) data with applications in neurosciences. A number of recent models from the neuroscience literature are especially vulnerable to noise, making structure-preserving smoothing an essential part in the analysis. Among the challenges are to establish connections between structural adaptive smoothing and diffusion within the Beltrami framework. The construction of connectivity maps integrates analysis of DWI and functional MRI data.

Three-dimensional imaging paves the way for quantitative and patient-specific medicine. An important subtask is image segmentation, the recognition and identification of anatomical objects, and their explicit geometric representation – providing the basis for numerical simulations in therapy and operation planning. Since manual segmentation is too costly for clinical routine, this process has to be automated. Automatic image segmentation, however, is

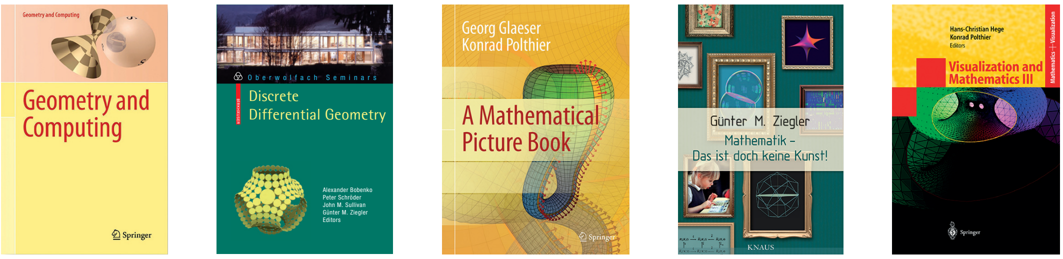

Figure 1. *Assortment of book series and books published with contributions of the application area visualization*

a long-standing problem: neither low-level nor high-level vision methods produce satisfactory results, except in special cases. The fundamental hypothesis is that this problem can be solved by incorporating a-priori knowledge about geometric-topological shape characteristics and image characteristics of the objects to be segmented in a so-called atlas. The general aim thus is to develop a mathematically sound, versatile, robust and automatic 3D image segmentation algorithm that utilizes such an atlas.

Mathematical visualization. Virtual reality means creating a three-dimensional scene in which the user feels immersed and with which he can interact to perform certain tasks. Virtual reality installations are widely used for scientific visualization and for industrial design. But additionally they are ideally suited for experimental mathematics and mathematical visualization too, and the potential of virtual reality for mathematics has barely been touched upon so far. Some time ago, Bill Thurston wrote convincingly about the great mental difference between imagining a three-dimensional object small enough to look at, and large enough to move through. He attributed many of his own unique insights into 3D geometry and topology to his adopting the latter viewpoint, of immersing himself in the 3D space. Within MATHEON we built immersive virtual-reality theaters at TU Berlin and ZIB to allow researches and the general public to immerse themselves into mathematical spaces.

Industrial cooperations. The research topics of this application area address key industrial problems in the areas of computer aided design, geometry processing and medial image processing. We developed novel algorithms in cooperation with various industrial partners including *Charité*, *DreamWorks*, *mental images* & *NVidia*, *DaimlerChrysler*, *Mercury Computer Systems*, *SciFace GmbH*, *Tebis*, *Teles Pri*, and several other international companies. For example, the mesh compression algorithm FreeLence (cf. Section F1) was incorporated into the rendering software of *mental images. Tebis* supported the development of new algorithms for feature detection and mesh segmentation which finally won a best paper award at the Eurographics conference 2004. *Mercury Computer Systems* supported research into fast algorithms for medical image analysis. *SciFace GmbH* in Paderborn, the producers of the computer algebra system MuPAD, supported the development of web-based mathematical visualization tools.

Mathematical software. Within this application area we contributed to several mathematical software systems:

Figure 2. *Videos MESH (*left) *and MathFilm Festival* (right)

- Amira (http://www.amira.com) is an advanced system for scientific and medical visualization, used in research and education. The base system of Amira is maintained by *FEI Visualization Sciences Group*. Modules for atlas-based image segmentation as well as modelling tools were developed and implemented.
- JavaView (http://www.javaview.de) based mathematical web-services are a featured web-Mathematica site of *Wolfram Research*. JavaView-Lite is used by *SciFace GmbH*, the developers of the computer algebra system MuPAD, as the geometry viewer of their Linux distribution. The JavaViewLib package is a *Maple Research* Powertool and the dataformat JVX is natively supported by Mathematica.
- EG-Models, the digital geometry model server at (http://www.eg-models.de), is a peer-refereed electronic journal initiated by M. Joswig and K. Polthier. It publishes geometric data sets which demonstrate new experimental research results, counterexamples or explicit geometric constructions. For the first time, the EG-Models server makes refereed experimental datasets widely available for mathematical research.
- jReality (http://www.jreality.de) is the java-based viewer developed as part of our virtual reality activities, intended for easily developing new interactive visualizations for research purposes in virtual reality environments and beyond.

Public events and outreach. International visibility has been achieved by organizing leading conferences and workshops such as “EuroVis 2009” and the “Symposium on Geometry Processing 2009”. We were also instrumental in shaping the Oberwolfach workshops on “Discrete Differential Geometry” in 2006 and 2009, and the Banff workshop on “Computational Mathematics of Discrete Surfaces” in 2009.

The outreach activities of MATHEON benefit strongly from the visual material we develop as part of our research. This was strikingly showcased during the nationwide German “Year of Mathematics” in 2008, in particular as a part of the Imaginary exhibition (see http://www.imaginary2008.de) and in the highly successful and visible “MathFilm Festival 2008” (see http://www.mathfilm2008.de). More than 130 000 visitors saw our contributions to the traveling Imaginary exhibit. The international MathFilm Festival 2008 with about 1000 film screenings in over 100 cities in Germany was among the most visible events of the year. Our logo design for the International Mathematical Union was accompanied with a short video about the Borromean rings. The first textbook on discrete differential geometry is a landmark

Figure 3. *Contributions to events and exhibitions: MathFilm Festival* (left)*, Imaginary 2008 (middle), and WELTWISSEN* (right)

for this novel research. The book “Bilder der Mathematik” plus its various translations provide an unprecedented visual insight into mathematics for both mathematicians and the general public. The mathematics video MESH has won international prizes since 2005 including the best science video prize at the New York International Film Festival as well as prizes in Australia, Brazil and Europe.