A Leap Forward: A User Study on Gestural Geometry Exploration

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ABSTRACT
Teaching mathematics in high school and university context often proves hard for both teachers and professors respectively. However, it can be supported by technology. Appliances for 3D digital setups are widely available. They have transcended their intended use as simple in- or output devices and nowadays also play a part in many artistic setups. Thus, they change the way we both perceive and create (digital) models. These changes have to be kept in mind when creating, working with, and presenting 3D art in a digital context.

In this paper, we examine the use of gesture-based controllers in the exploration of mathematical content. A user study was conducted as part of the scientific art and education exhibition "Long Night of Science". To validate the results, a control group was presented with the same questionnaire and physical models of the mathematical objects (instead of the controller) were used. The participants of the study rated the controllers or the physical models respectively by their individually felt intuitiveness and influence on the perception of the underlying mathematical content. From the data obtained, a connection between the intuitiveness of the controller and a positive influence on the perception of the presented mathematics is shown.

KEYWORDS
User Experience; Gestural Control; Gestures; User Study

1. Introduction

Conveying knowledge is a tough job. Teachers will agree with this statement. It holds especially for those subjects that are not very popular with the students, for example mathematics. Therefore, it is a long-standing question how to motivate students and how to make learning a fun experience.

Apart from this fundamental question, in recent years there was a rapid development in controller technology. For instance, mobile phones are controlled to an increasing extend with voice commands. Also, Nintendo and Microsoft introduced completely new controller concepts to the living rooms around the world when launching the Wii and Kinect respectively. To use these is more intuitive and imposes less barriers on many users than standard input devices like mouse and keyboard do.

In high schools, computers are available on a large scale for many students. When using tools in the classroom, it is important to have as many students work simultane-

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ously as possible. Storing enough physical models of for example the platonic solids for a whole class takes up considerable space. Besides, the models need to be maintained, handed out at the beginning of class, and collected at the end of the session. Equipping the students with new controlling devices for the computers reduces this effort. Furthermore, these devices are not limited to a single topic or subject and can be used throughout the whole curriculum. Therefore, they are worthwhile to be investigated in a didactic context.

In this work, we explore the use of a gesture-based controller in the context of mathematical didactics. We set up an interactive exhibit during an annual scientific art and education exhibition, the “Long Night of Science”. During and after this exhibition, we conducted a study to show that the individually felt intuitiveness of the considered controller has a significant effect on the individual perception of the mathematical content. To validate our findings, we compare the new gesture-based controlling device to old-fashioned physical models of mathematical objects with a control group. In the final section, we give an interpretation of the results obtained from the study and elaborate on both the limitations of our experiment as well as on future research directions. Furthermore, we position our results in the context of (mathematically inspired) art.

2. Related Work

2.1. The perception and intuitiveness of 3D Systems and their usage in the Arts

Nowadays, there is an increasing use of 3D computer graphics in all areas of everyday life. In this section we will briefly review relevant aspects of the (math) art literature and showcase how 3D systems are used in art processes already nowadays. Given the topic of this paper, we focus on the differences of perception that are introduced by technology.

One example as to how digital appliances can alter or even control the process of physical creation is presented in a paper by Zoran and Paradiso [31]. Here, the considered design process starts in the digital realm, by picking or creating a 3D digital model. Then, working with a block of material, the user has a milling device which he or she can use as a sculpting tool to create the desired geometry representing the chosen 3D model. The program always closely tracks the user’s motions, intervening only if the structural integrity of the created object is at stake. Thus, the user will create the desired form under guidance of the digital program while still maintaining an artistic degree of freedom.

In a way, the approach presented in the present paper turns the process of Zoran and Paradiso [31] around. While in their approach, the digital model and program interferes with the manual creative sculpting process, in our setup, the user can manually interact with and alter the digital realm. The performed gestures are translated into the program’s commands to provoke certain actions. The same general concept is followed by Zamit and Munoz [30] within the context of vehicle design. They conclude that traditional clay modeling of cars is not necessary any more as stylus based polygonal modeling has reached the required resolution to generate models of high quality. In particular during the early design stage where many ideas enter, digital processes can help with easier exchange, alteration, and iteration of the models.

Despite their technical progression and improvements, digital 3D technologies also have to provide an intuitive user interface. Ye and Campbell [29] state that "conventional computer aided design (CAD) systems have not provided enough support for conceptual design partly due to the lack of natural and intuitive human–computer interaction." In their paper, they develop a conceptual design system to integrate VR-based interfaces into design applications. We see from this example that not only the possibilities, but also the challenges increase with more and more integrated use of 3D technology. Namely, the interface has to be as intuitive as the traditional physical interfaces which users are used to. This observation largely motivates one of our questionnaire items, see Section 6.

Turning to arts, the study of Lee et al. [12] explores the roles of cognitive evaluation using a 3D sculpture tool and its relationship with design thinking types. They find that "the types of cognitive evaluation and their roles when using the 3D sculpture tool (are) different, according to the design thinking type." Thus, particularly in creative spaces, digital design environments can foster some qualities and hinder others. Furthermore, the study of Creed [5] focuses on a very special target group: disabled visual artists. The author explores how digital tools are used by this group in order to create art. The study clearly states that "novel forms of technology (...) present new creative opportunities." These findings motivate the research presented in this paper.

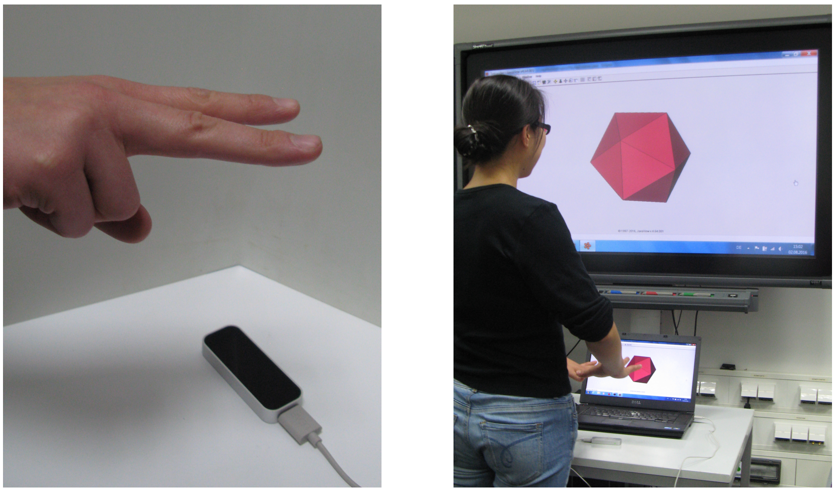
Finally, digital sculpting tools are also used in the context of mathematical art. A collection of examples can be found in a book by Henry Segerman [27]. It showcases how the precision of the digital design process can be mixed with 3D printers to provide a multitude of new experiences for both mathematical concepts and mathematical arts. Another project by Henry Segerman—bringing the audience into the 3D realm—is presented in Stephen Ornes' book [20]. In the presented virtual reality installation "Monkey See, Monkey Do", the users can explore four-dimensional geometries solely by the means of moving in virtual reality. This kind of interplay between the physical entity of the user and the given virtual mathematical models is what this paper also aims for.

2.2. 3D Systems and their use in Schools

Despite the wide spread of 3D computer graphics, most 3D navigation systems are still based on the classical Arcball model by Ken Shoemake [26]. Although this model is very helpful, it suffers from the translation of 2D input (like mouse coordinates) to 3D navigation. Therefore, current research focuses more and more on gesture-based systems with 3D input. Consider for example the work of Bailly et al. [2], where the authors use gestures for the control of mobile devices or the paper by Kratz et al. [11] where 3D rotation is implemented. The latter shows advantages over the classical Arcball model.

In this project, we use the *LMC* (Leap Motion™ Controller, by Leap Motion™ Inc.) for tracking hand gestures. It was already successfully used for example in a work about Australian sign language recognition, see [23]. Many more possible applications can be found in an online project repository, see [13] and cf. Section 10 for a discussion of applications to art. The *Leap Motion™ App Store* provides a broad range of apps. Many of them are educational, yet only a few deal with mathematics, for example [24]. Like our project, it can be used to observe and manipulate geometrical figures in 3D space.

While critics are afraid that the use of technology affects the students' performance in mathematics negatively, several studies find the opposite to be true. In the work



(a) The Leap Motion$^{\text{TM}}$ Controller by Leap Motion$^{\text{TM}}$ Inc.

(b) A student using the Leap Motion$^{\text{TM}}$ Controller to explore an icosahedron in the *JavaView* geometry viewer.

Figure 1. Usage of the Leap Motion$^{\text{TM}}$ Controller.

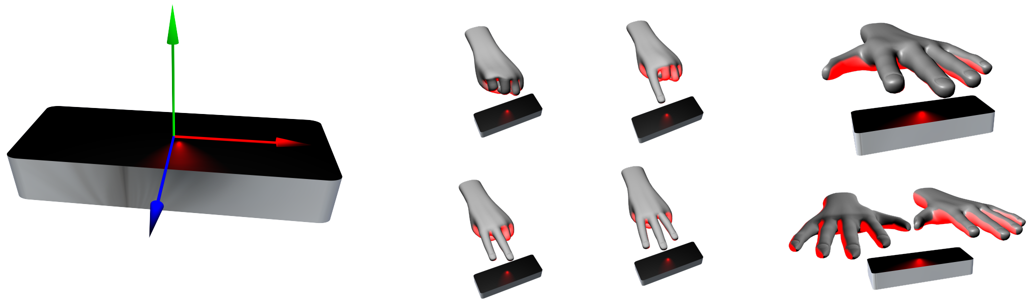
of Olive and Makar [19], the authors argue that the use of technology can strengthen the connection between mathematical knowledge and practices. Furthermore, in a monograph by Scouter [25], teachers report that “using technology would make their students more motivated.”

The presented findings motivate us to consider 3D gesture control in the context of geometry exploration. In the following, we present our corresponding experiment setup.

3. Used Hard- and Software

For our project, we used the LMC, a computer hardware sensor device supporting hand and finger motions in 3D space as input. See Figure 1a for an image of the controller. Unlike the sensor bars of the Kinect, the LMC uses infrared optics and a camera instead of depth sensors. Hidden under the glossy black panel on top, the sensors are directed along the vertical axis, a right-handed Cartesian coordinate system is employed, see Figure 2a. The LMC can track up to ten fingers with high precision as well as equally fast latency and reports discrete positions and motion. Due to the fine-tuned motion control, it provides new possibilities for computational control.

The Leap Motion$^{\text{TM}}$ Inc. company provides the *Leap Motion^{TM} SDK*, so one can build new applications using existing libraries, examples, and documentations. The application programming interface returns the tracking data as frames which contain lists of tracked entities (i.e. hands, fingers) along with objects representing recognized gestures and motion related factors. Furthermore, the API reference provides details on all available classes. Though the initial software library is still relatively limited, the interest from developers grows. One goal of this paper is to showcase why the LMC should not be underestimated and in what fields it has great potential. Several great applications can be found in the gallery [13]. Aside from games, a major category of available apps is educational, for example frog dissection or exploration of a human



(a) Cartesian coordinate system employed by the LMC.

(b) The supported gestures in our program: fist; one, two, three, and ten fingers stretched out; five fingers for swiping.

Figure 2. Coordinate system and supported gestures of the LMC.

skull. The improved spatial control as a result of the additional vertical axis, compared to a traditional mouse, allows a more profound understanding and easier manipulation of 3D digital objects.

As a consequence of the simple utilization, the LMC can help especially students to deal with mathematics in a more fascinating and motivating way and thus decrease the objection against such an important scientific area. For this purpose, we used the software *JavaView*, see [10], as a basis. *JavaView* is a 3D geometry viewer and a mathematical visualization software. Its library contains sophisticated algorithms for geometric modeling, surface optimization, and visualization. See Figure 1b for an example of the usage of the LMC with *JavaView*.

4. Description of the Test Setup

The connection of the visualization software and the LMC is created by assigning gestures recognized by the controller to different functions in *JavaView*. We restrict to the six simple gestures shown in Figure 2b and an additional swipe motion. Here, we describe the different gestures and their respective effects in the visualization software *JavaView*.

By moving a fist up or down, the user can magnify or shrink displayed objects. The LMC internally processes frames. In order to achieve a fluent zoom, the height difference between the current and the last frame is compared. This comparison is performed as long as the hand moves above the controller. The high frame rate, which is adapted to the system performance, ensures this gesture to be very robust despite the movement involved. Additionally, we set upper and lower bounds for the zoom to ensure the geometry to remain visible at any time.

Holding one to three fingers over the LMC results in a rotation of the geometry around the horizontal, vertical, and depth axis of the imposed coordinate system respectively, as illustrated in Figure 2a. So far, in our implementation, only clockwise rotations are possible. Spreading all ten fingers over the controller activates the explode workshop of *JavaView*, as shown in Figure 3, followed by an automatic reset after a few seconds. These gestures can easily be learned, as they do not involve any movement.

After launching the program, a tetrahedron is displayed by default. Swipe gestures to the left and right, with five fingers stretched out, enable the user to switch between the five platonic solids forward and backward respectively. There are predefined methods in the Leap Motion$^{\text{TM}}$ library which can detect the direction of certain gestures within the Leap Motion$^{\text{TM}}$ coordinate system. However, the detection is harder than for the other gestures and thus swipes have not been reported as robustly as the stretched out fingers or the fist, which was also remarked by the study participants (see Section 9). Swipes into other directions beside left/right are ignored at this stage.

We have confined ourselves to this set of gestures because they are easy to learn and to remember, as well as robustly detected by the controller. This enables the participants to experience the new controller quickly without a lengthy instruction phase. Note that the LMC replaces a mouse and keyboard combination which is usually used to control *JavaView*. The functionality has not been adapted to the alternative controlling device.

Higher effort can be put into both the controls and the output for an even better user experience, c.f. [24]. However, this comes with longer initial training time, which was not desired in the present study.

5. Hypotheses

In the following, we will use the two terms *usage* and *perception*. By *usage* we denote the dimension explored by the second question of the questionnaire presented in Figure 4b ("How intuitive did it feel to use the Leap Motion$^{\text{TM}}$ Controller?"). By asking the participants about their feelings on the intuitiveness of the controller, we therefore do not measure the actual usage in this context but rather the impression of the participants on their respective individual usage.

Similarly, *perception* denotes the dimension explored by the third question of the questionnaire presented in Figure 4b ("How did the use of the Leap Motion$^{\text{TM}}$ Controller as replacement of the mouse alter your perception of the geometric bodies?"). Once more, the actual perception of the mathematical contents is not measured by this question. For this, one or more questions on the presented geometry would have to be posed, see Section 10. Thus, the questionnaire item *perception* measures the individually felt effect of the controller on the perception of the mathematical content.

In order to distinguish between the terms as defined above from their common meaning, we will put them in italics, i.e. *usage* and *perception*, whenever describing individual impressions and without italics when using the terms in their respective common sense. Having defined these two central notions we turn to the formulation of hypotheses based on the data obtained from our questionnaire.

Based on the work of [19] and [25] we conjecture that the intuitiveness of usage of the device is correlated with the perception of the transmitted information. Since didactics in general aims for the taught contents to be perceived positively, we further restrict

Figure 3. An illustration of the explode workshop of *JavaView* (frames of the explosion from left to right).

the above conjectured correlation to a directed correlation of *usage* on *perception* (H1).

Furthermore, e.g. Berenbaum et al. [3] found that boys tend to focus more on movement based play. Thus, they possibly develop a faster understanding for spatially complex environments. However, a gender distinction like this is highly controversial. In a large-scale review paper by Coluccia and Louse [4], the issue is found to be more subtle. As a contribution to this discussion and following the more and more common perception that male participants do not have a higher affinity to technology, we conjecture that there is no significant effect of gender on neither *usage* (H2) nor *perception* (H3).

Finally, in any schooling context, the LMC would compete with old-fashioned, long established physical models used to convey mathematical content. Since the LMC is easy and intuitive to use, while the controlled program is equipped with a multitude of digital features not present in physical models, we conjecture that the LMC will perform as good as physical models in terms of *usage* (H4) and *perception* (H5).

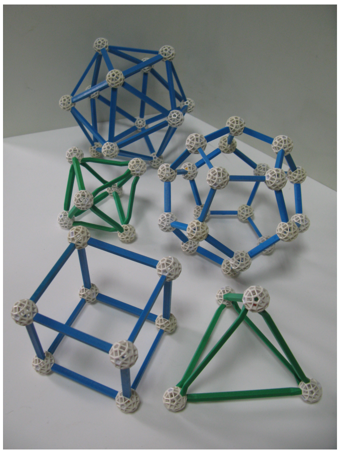
6. Procedure and Testing Environment

The testing was done on three different occasions. Two groups tested the LMC while a third control group was given physical models of the platonic solids instead of digital models. Details on the three groups are given in Section 7.

First, the LMC was presented during the "Long Night of Science", which is an annual science education event where scientific institutes in Berlin, Germany, introduce their research to the public. This well-established event has proven to appeal to large audiences of all ages willing to educate themselves. Beside a controllable digital kaleidoscope and the LMC as interactive parts, our exhibition also included poster presentations on the latest research topics of the work group "Mathematical Geometry Processing (FU Berlin)" and a six feet tall *Zometool*™ model of an omnitruncated hyperdodecahedron. However, visitors showed particular interest in operating the geometry software *JavaView* with the LMC. Especially children, which are the target group of the innovation, enjoyed exploring the platonic solids in this new way. The first group of study participants comprises solely of visitors to the "Long Night of Science". The second group was given opportunity to try out the LMC during a guided tour through the work group of "Mathematical Geometry Processing (FU Berlin)". Finally, a third group was confronted with physical models. This group acts as control group in the presented study. It consists of students of a Berlin high school who were shown the models in their classroom.

For all three groups, the testing procedure was exactly the same. The participants were told several facts about the platonic solids by an instructor. This took about five minutes. Then they were either instructed in the usage of the LMC or were shown the Zometool™ models and taught how those are built. The instructions were given in small groups of up to six participants. Then, they could individually use the LMC or the physical models as long as they would like to (about nine minutes on average). After using the controller or the models, they were asked to fill out the questionnaire shown in Figure 4b. For those participants using physical models, each occurrence of "Leap Motion™ Controller" was replaced by "Zometool™ models". The questions on age and improvements allowed free answers, all other questions provided boxes with prescribed answers as indicated.

Note that the survey with the questionnaire shown in Figure 4b was originally conducted in German. Thus, the questionnaire as given in this paper is a translated



(a) The physical Zometool$^{\text{TM}}$ models as presented to the control group.

Gender:	☐ male / ☐ female
Age:	-----
Was today the first time for you to use the Leap Motion$^{\text{TM}}$ Controller?	☐ Yes ☐ No
How intuitive did it feel to use the Leap Motion$^{\text{TM}}$ Controller?	☐ very intuitive ☐ intuitive ☐ unintuitive ☐ very unintuitive
How did the use of the Leap Motion$^{\text{TM}}$ Controller as replacement of the mouse alter your perception of the geometric bodies?	☐ very positively ☐ positively ☐ no alteration ☐ negatively ☐ very negatively
Do you have improvement proposals or ideas for the usage of the Leap Motion$^{\text{TM}}$ Controller?	-----

(b) The questionnaire that was handed out to the participants of the presented study.

Figure 4. Physical models for the control group and the questionnaire used in the study.

version. The important words “intuitive” and “perception” are semantically equivalent to the German correspondences “intuitiv” and “Wahrnehmung”.

7. Participants

Two groups of participants were recruited for testing the LMC. The first group consisted of visitors to the “Long Night of Science”. The second group consisted of current high school students and young Bachelor students visiting the work group to see elements of the former mentioned science exhibition. Altogether, the number of participants was $N = 41$. There were 18 female and 23 male participants, aged 8 to 50 ($median = 17$, $sd = 9.597$), with one participant not giving her or his age. In total, five participants already had experience with the LMC.

Concerning the control group, it consisted of students of a Berlin high school. The number of participants in the control group was $N = 18$. There were 11 female and 7 male participants, aged 13 to 15 ($median = 13$, $sd = 0.598$). One participant already had experience with the LMC. See Section 10 for a discussion of limitations of the study resulting from the composition of the participant groups as presented here.

8. Results

Before analyzing the data, the four categories concerning the *usage* as well as the five categories concerning the *perception* were encoded according to the scheme in Table 1.

Note at this point that both variables *usage* and *perception* are ordinal variables, where *usage* does not have a neutral answer element to counteract the error of central tendency. Hence, the encoding “0” is left out for the *usage* values.

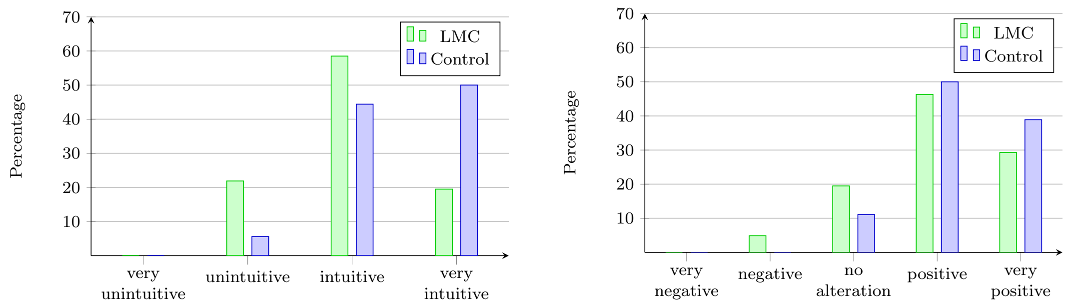
Given the encoding from Table 1, for the LMC group, *usage* has an arithmetic mean of $m = 0.756$ ($Q1 = 1$, $median = 1$, $Q3 = 1$, $sd = 1.007$), while *perception* has an arithmetic mean of $m = 1$ ($Q1 = 1$, $median = 1$, $Q3 = 2$, $sd = 0.826$). The relative self-information by Shannon is comparably high for both *usage* ($H = 0.696$)

Table 1. Encoding of the ordinal variables *usage* and *perception*.

Answer (*usage*)	Answer (*perception*)	Encoding
very intuitive	very positive	2
intuitive	positive	1
-	no alteration	0
unintuitive	negative	-1
very unintuitive	very negative	-2

and *perception* ($H = 0.734$).

Again, with the encoding from Table 1, for the control group, *usage* has an arithmetic mean of $m = 1.389$ ($Q1 = 1$, $median = 1.5$, $Q3 = 2$, $sd = 0.756$), while *perception* has an arithmetic mean of $m = 1.278$ ($Q1 = 1$, $median = 1$, $Q3 = 2$, $sd = 0.650$). The relative self-information by Shannon is comparably high for both *usage* ($H = 0.626$) and *perception* ($H = 0.595$). Refer to the histograms in Figure 5 for further information on the distribution of values for both variables and a visual comparison of the distribution in the LMC group and the control group.



(a) Histogram on the answers concerning *usage*. (b) Histogram on the answers concerning *perception*.

Figure 5. Histograms for the answers on *usage* and *perception*.

8.1. H1: Correlation of Usage and Perception

In order to quantify the correlation between the variables *usage* and *perception*, Goodman's and Kruskal's gamma was computed, see [7, p. 545]. It gives an undirected measure of rank correlation and is therefore recommended for categorical data with ordered answer categories. Comparing concordant and non-concordant pairs, we find a correlation of $\hat{\gamma} = 0.371$. This indicates that the *perception* has other influences aside from *usage*. However, it also indicates a weak linear relationship between both.

Given the encouraging value of $\hat{\gamma}$, to further investigate the influence of *usage (U)* on *perception (P)*, we compute a linear least squares regression see [7, p. 595 ff.]: $P = b_0 + b_1 \cdot U$. Using the statistics software *R* and its linear model, we obtain $P = 0.799 + 0.265 \cdot U + e$ with a determination coefficient $R^2 = 0.104$ and some minor error term e. Since we estimate *perception* on a single predictor, R^2 can be interpreted as percentage. In other words 10.4% of the variance in *perception* can be explained by the variance of *usage*. The estimated residual variance is $\delta_\varepsilon^2 = 0.643$ and the squared standard error of our correlation term b_1 is $\delta_{b_1}^2 = 0.015$. The two-sided 95%-confidence interval is given as $b_1 \pm t(1 - \frac{\alpha}{2}; 39) \cdot \delta_{b_1} = 0.265 \pm 2.021 \cdot 0.124$. Since 0 is not included, the regression weight is statistically different from 0 by $p = 0.03977$.

8.2. H2 and H3: Effects of Gender on Usage and Perception

In order to reveal any effects of gender on either of the variables *usage* or *perception*, a Wilcox-Rank-Sum-Test see [7, p. 343 ff.] is performed. It is suitable to compare two independent samples in which the investigated categorical property does not necessarily follow the normal distribution and where the samples are too small to perform a t-test. It indicates that ranks of female participants do not significantly differ from the ranks of male participants concerning both *usage* ($z = 0.583$, $p = 0.05$) and *perception* ($z = 0.439$, $p = 0.05$).

8.3. H4 and H5: Comparison between the LMC Group and the Control Group

In order to bring the results on the LMC in line with other available mathematical didactic tools, a comparison with a control group and physical models was performed. Once more, we employ the Wilcox-Rank-Sum-Test see [7, p. 343 ff.] in order to determine whether the values for *usage* or *perception* differ significantly between the LMC group and the control group. The test indicates that the values for *usage* differ significantly ($z = 2.287$, $p = 0.05$) with a confidence interval of $[-1.96, +1.96]$. We compute the corresponding effect size ($\hat{\theta} = 0.315$, $p = 0.05$). The two-sided confidence interval for the effect size according to [18] is $[+0.193, +0.476]$. Assuming the null hypothesis, one would expect $\theta_0 = 0.5$. This value is not in the confidence interval, therefore, the two samples differ with significant effect size.

Concerning the *perception*, the ranks of the LMC group and the control group do not significantly differ from each other ($z = 1.041$, $p = 0.05$).

9. Interpretation

The age of the participants in this study fits the later target group of the project very well, most of them being high school or first to second year university students. In particular the control group was comprised solely of high school students. Our first hypothesis H1 was confirmed by the statistical analysis given above. Hence, the *usage* of a rather intuitive device in the setting of exploring mathematics has a positive outcome on the *perception* of the (mathematical) models and concepts.

Furthermore, our data did not reveal a significant difference between male and female participants. Although some parts of the literature suggest such effects, we cannot concur.

Five participants had already used the LMC before. Their ratings on the controller were not uniform (*usage*: 2 × "unintuitive", 2 × "intuitive", 1 × "very intuitive"; influence: 2 × "no alteration", 1 × "positive", 2 × "very positive"). Although the low number does not permit a full statistical comparison, we are confident that these users do not significantly differ from first-time users.

In the questionnaire, the participants were asked to improvement proposals. Five participants suggested more features, such as more complex gestures for easier 3D navigation or counter-clockwise rotations. Six participants asked for a more robust detection of their gestures and a better definition of the space in which gestures are processed by the controller. Surprisingly, four of them still rated *perception* as "(very) positive".

Finally, the results of the control group indicate that the LMC performs up to par with traditional, physical classroom models when compared on the property of influence on the mathematical content that is transported. However, traditional models were ranked significantly better in terms of *usage*. Given the improvement proposals discussed above, we assume that technical improvements can diminish or even eliminate the difference.

10. Conclusion, Limitations, and Future Work

We have shown in our study that the usage of an intuitive input device, like the gesture detecting LMC, has statistically significant positive influence on the individual impression of perception of the presented mathematics. This is in line with the findings of Olive and Makar [19] and those of Scouter [25]. No gender specific differences were found, which is in contrast to some results presented in the study of Berenbaum et al. [3].

The results show that a new controller technology, namely gesture-based controllers, has to be considered as possible addition to the classroom setting. Controllers become more and more affordable and the needed computers are available in many schools. As the content is not coupled to the device—as it is the case for physical models—new controlling techniques can be used in several different subjects and settings. Possible applications lie in geography, introducing the concept of continental drift [8], in arts, where users can explore the possibilities of pottery [1], or biology, where a human skull can be explored, dissected, and assembled [6].

In Section 2.1, we have discussed in what ways our setup is motivated by current developments in the usage of 3D exploration tools in the artistic context. As the work of Henry Segerman [27], our installation enables users to have a different experience with the presented mathematical content than they would have with simple physical models. Similar to the report of Creed [5], we provide very low-demanding access to the installation, making it available for users of all ages and abilities. While the results of this paper are mainly rooted in the context of mathematics, both the works of Segerman [20, 27] and the LMC pottery application [1] already hint at applications of gesture-based controllers in arts. In fact, several corresponding applications have been realized with the LMC. A general overview on the art-related projects utilizing the LMC can be found in the corresponding category of the LMC developer blog [14]. Additionally, the Leap Motion™Gallery [13] features several art-related projects to explore. These include creativity applications for sketching [9] or painting [21] directly in 3D space. If an object is given as 3D mesh, it can be explored and manipulated directly by hand gestures [16]. Furthermore, the beautiful patterns of particle simulations can be explored and altered [17, 22]. As final examples, even musical compositions can be derived in virtual 3D space [15, 28]. For those who want to explore gesture-based control themselves, we recommend the Game Science Center Berlin[1] to explore projects as the above mentioned first hand.

As discussed in the interpretation, Section 9, gesture detection needs to be robust and more intuitive gestures than the ones used in our experiments have to be implemented. As these are computationally more complex, such implementation is left as

[1] http://www.gamesciencecenter.de/

future work. In particular, these future experiments will clarify whether the differences between the two groups will be even greater as soon as more natural gestures are available.

The presented study suffers from three limitations. First, the number of participants is comparably low and the age-range is very different for the participants using the LMC and the control group. A broader study is desirable to further validate our findings. Second, the two approaches presented (technological presentation and physical handling of the platonic solids) are very different in their nature. To better locate the found effects, we propose to include further comparison items in a follow-up study. Namely, to better distinguish the gesture-based control from other control inputs, it should be compared to mouse-, keyboard-, and touchscreen inputs. Thus, the presentation would always be digital and better comparable. In particular, the set of gestures should be altered to even better mimic the handling of physical models. A third and final limitation concerns the physical models presented. Their facets are not present, but only the edges and vertices are given. Therefore, they allow for more exploration than the platonic solids shown in our digital setup, see Figure 1b. In future evaluations, these presentation have to be unified.

While the current questionnaire as given in Figure 4b is designed to measure the impression of the users on usage and perception, it is reasonable to extend the questionnaire also by mathematical questions to objectively evaluate the users' understanding of the underlying geometries. This was undesired in our context, as the questionnaire was handed out during a science education event for the public. Testing people there on their geometry knowledge would have gone against our goal of teaching them to lose the fear of contact to math and might have jeopardized the whole study. A next step in our work will consist of these improvements to the setup.

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