Simplex

Step 1

Start at the basic feasible solution $v = (0,0)$ defined by the basis $I = \{IV, V\}$

$$A_I = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Compute u

$u^t = c^t A_I^{-1}$

$$A_I^{-1} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$u^T = (1,3) \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = (-1,-3)$$

both entries are < 0 so we can chose one arbitrarily. Lets chose i=2, which means that V leaves the Base I

Compute d

$d = -A_I^{-1} e_i$

$$d = -1 \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ note that this means to move along the edge defined by IV

But how far can we move along this edge?
Until we hit the next vertex!

This means that one of the other inequalities not in I is fulfilled with equality and forms a vertex together with $I \setminus$ {V}= IV

So for each row l where $A_{l*} d > 0$ we need to find the value λ such that:
$A_{l*}(v + \lambda d) = b_l$

Of all these λ we select the minimal one, and the index k for which this λ was attained enters out new base.

We get the following values for λ:

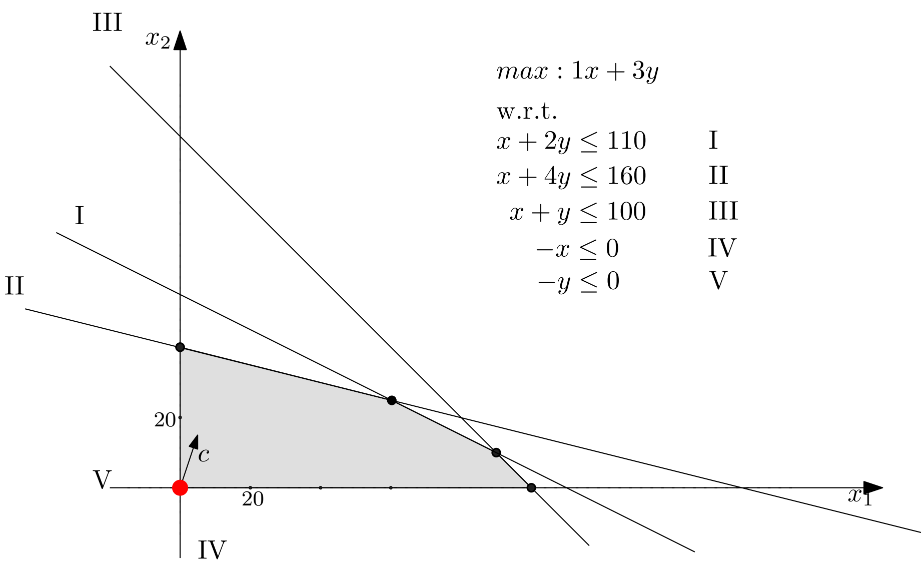
$$\lambda_1 = \frac{110-0}{(1,2)\begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \frac{110}{2} = 55$$

$$\lambda_2 = \frac{160-0}{(1,4)\begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \frac{160}{4} = 40$$

$$\lambda_3 = \frac{100-0}{(1,1)\begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \frac{100}{1} = 100$$

The minimum value for λ is 40 and is attained for index $k = 2$ and therefore the corresponding row (II) enters the base:
I={II, IV}
The corresponding basic feasible solution is:

$$v = v + \lambda d = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 40 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 40 \end{pmatrix}$$



$max : 1x + 3y$

w.r.t.

$x + 2y \leq 110$	I
$x + 4y \leq 160$	II
$x + y \leq 100$	III
$-x \leq 0$	IV
$-y \leq 0$	V

Step 2

$$A_I = \begin{pmatrix} 1 & 4 \\ -1 & 0 \end{pmatrix}$$

Compute u

$u^t = c^t A_I^{-1}$

$$A_I^{-1} = \begin{pmatrix} 0 & -1 \\ 1/4 & 1/4 \end{pmatrix}$$

$$u^T = (1,3)\begin{pmatrix} 0 & -1 \\ 1/4 & 1/4 \end{pmatrix} = (3/4, -1/4)$$

only the second entry is negative, so take $i = 2$, This means that IV leaves the base I.

Compute d

$d = -A_I^{-1} e_i$

$d = -1\begin{pmatrix} 0 & -1 \\ 1/4 & 1/4 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1/4 \end{pmatrix}$ note that this means to move along the edge defined by II

But how far can we move along this edge?
Until we hit the next vertex!

This means that one of the other inequalities not in I is fulfilled with equality and forms a vertex together with $I\setminus$ {IV}= II

So for each row l where $A_{l*}d > 0$ we need to find the value λ such that:
$A_{l*}(v + \lambda d) = b_l$

Of all these λ we select the minimal one, and the index k for which this λ was attained enters out new base.

We get the following values for λ:

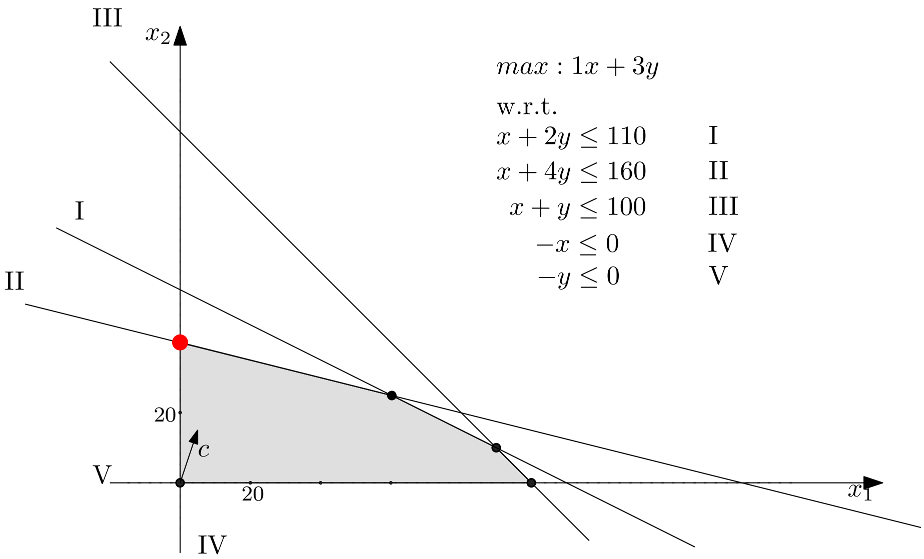
$$\lambda_1 = \frac{110-(1,2)(0,40)^T}{(1,2)\begin{pmatrix} 1 \\ -1/4 \end{pmatrix}} = \frac{110-80}{1/2} = 60$$

$$\lambda_3 = \frac{100-(1,1)(0,40)^T}{(1,1)\begin{pmatrix} 1 \\ -1/4 \end{pmatrix}} = \frac{60}{3/4} = 80$$

$$\lambda_5 = \frac{0-(0,-1)(0,40)^T}{(0,-1)\begin{pmatrix} 1 \\ -1/4 \end{pmatrix}} = \frac{40}{1/4} = 160$$

The minimum value for λ is 60 and is attained for index $k = 1$ and therefore the corresponding row (I) enters the base:
I={I,II}
The corresponding basic feasible solution is:

$$v = v + \lambda d = \begin{pmatrix} 0 \\ 40 \end{pmatrix} + 60\begin{pmatrix} 1 \\ -1/4 \end{pmatrix} = \begin{pmatrix} 60 \\ 25 \end{pmatrix}$$



Step 3

$$A_I = \begin{pmatrix} 1 & 2 \\ 1 & 4 \end{pmatrix}$$

Compute u
$u^t = c^t A_I^{-1}$

$$A_I^{-1} = \begin{pmatrix} 2 & -1 \\ -1/2 & 1/2 \end{pmatrix}$$

$$u^T = (1, 3) \begin{pmatrix} 2 & -1 \\ -1/2 & 1/2 \end{pmatrix} = (1/2, 1/2)$$

Both entries of u are nonnegative. Therefore the actual vertex $v = (60, 25)$ is an optimal solution of our problem.

DONE!