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Discrete Mathematics for Bioinformatics (P1)

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Exercises 5

1. **Prove the two Lemma from the lecture**

   Lemma 1. *Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then*

   - *$P_{\mathcal{R}}(G)$ is full-dimensional and*
   - *the inequality $x_i \leq 1$ is facet-defining iff there is no $e_j \in E$ in conflict with e_i.*

   Lemma 2. *Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then*

   - *The inequality $x_i \geq 0$ is facet-defining iff e_i is not contained in an interaction match.*
   - *For each interaction match $m_{i,j}$ the inequality $x_{i,j} \geq 0$ is facet-defining.*

   Hint: find a sufficient number of affinely independent vectors.

2. **Landau Symbols**

   Show the following:

   (a) $\forall k, l \in \mathbb{Z}. k > l : n^l = o(n^k)$

   (b) $\forall k, l \in \mathbb{N}. k > l : n^k + n^l = \Theta(n^k)$

   (c) $f = O(2^n) \Leftrightarrow f = 2^{O(n)}$?

3. **Average Case Analysis of Quicksort**

   Show that the average number of comparisons during a Quicksort is $O(n \log n)$. You can assume that each element has the same probability to be chosen as pivot element and all partial sequences generated during every divide-step are also uniformly distributed.

   Further you might use the following identity:

   $H_n := \sum_{j=1}^{n} \frac{1}{j} = \ln n + C + o(1),\ C \approx 0.57721(Euler - Mascheroni - Constant)$

4. **Amortized Costs**

   Let the costs for updating a binary counter be the number of changed bits.

   (a) Given a counter that implements only the operation 'increment'. Compute the cost of a single operation in worst, best and average case.

   (b) Apply a suitable potential function ϕ and compute the amortized cost.

   (c) How do the costs change if the counter also supports 'decrement'?

   (d) What ist the total actual cost if the counter does not start at zero?