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## Discrete Mathematics for Bioinformatics (P1)

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### Exercises 4

1. *Lagrangean Relaxation*

Consider the following problem

$$\begin{array}{llll} \min & 3x_1 & - & x_2 \\ \text{w.r.t.} & x_1 & - & x_2 \geq -1 \\ & -x_1 & + & 2x_2 \leq 5 \\ & 3x_1 & + & 2x_2 \geq 3 \\ & 6x_1 & + & x_2 \leq 15 \\ & & & x_1, x_2 \geq 0 \\ & & & x_1, x_2 \in \mathbb{Z} \end{array}$$

- Draw the corresponding polytope and determine graphically the optimal solution  $Z_{IP}$  of the original problem and  $Z_{LP}$ , the solution of the LP-relaxation.
- Now apply lagrangean relaxation by relaxing the first inequality. Draw the polytope of the relaxed ILP. Determine the set  $X$  of feasible solutions for the relaxed problem.
- The new objective function is then:

$$Z(P) = \min_{(x_1, x_2) \in X} 3x_1 - x_2 + p(-1 - x_1 + x_2)$$

Calculate  $Z_D = \max_{p \geq 0} Z(p)$  and compare this value to  $Z_{IP}$  and  $Z_{LP}$ . (To obtain  $Z_D$ , draw the graphs of the function  $f(p) = 3x_1 - x_2 + p(-1 - x_1 + x_2)$  for all  $(x_1, x_2) \in X$ .)

- repeat a-c for the objective functions  $-x_1 + x_2$  and  $-x_1 - x_2$  and compare  $Z_{LP}$ ,  $Z_D$ , and  $Z_{IP}$ .

2. *Prove Lemma*

Prove Lemma 1 (see script page 4001) stating that (in case of a minimization problem) if  $\lambda \geq 0$ , then  $Z(\lambda) \leq Z_{IP}$ , where  $Z_{IP}$  is the optimal value of an original ILP and  $Z(\lambda)$  is the optimal value of the relaxed problem for a given value of the Lagrangean multiplier  $\lambda$ .

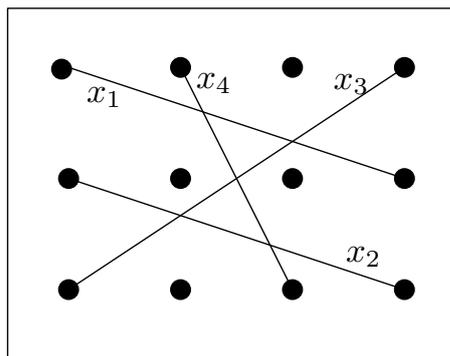
3. *Branch and Cut*

For the following exercises, you will probably need an LP solver – as for example ‘LP-Solve’.

Apply the cutting plane method to compute an optimal alignment of two sequences „ACC“ and „ACA“ where a match scores 1 and a mismatch or gap scores 0:

- (a) Draw the alignment graph, the conflict graph, and the pair graph.
- (b) Now start with the trivial (relaxed) LP and add successively clique inequalities which you can find on the longest paths in the pair graph that is labeled with the solution of the last step. Repeat this until you get the optimal alignment.

4. Given the following alignment graph:



All edges have weight 1.

- (a) Try to solve the alignment problem by using branch-and-cut: Add mixed cycle inequalities (see the ‘shortest path’ method in the script, page 18) to the corresponding (relaxed) LP. Can you reach an optimal solution for the ILP without branching?
- (b) Now use branching to solve the problem.
- (c) Instead of branching, just add the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 2$$

Can you solve the ILP now?

- (d) Prove that the inequality in (c) is facet-defining.