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Exercises 11

1. **Tree Decompositions**

   Prove the following theorems from the script:

   (a) Let $G = (V, E)$ be a graph, T be a tree decomposition of G, and (x, y) an edge in T. The deletion of (x, y) divides T into two components X and Y. Let V_x and V_y be the 'pieces' of x and y, respectively. Then deleting the set $V_x \cap V_y$ from V disconnects G into the two subgraphs $G_X - (V_x \cap V_y)$ and $G_Y - (V_x \cap V_y)$.
   (G_M for $M = X, Y$ is the subgraph of G that consists of all nodes in the 'pieces' of M.)

   (b) A connected graph G has tree-width 1 if and only if it is a tree. (Hint: Use (a) to create a contradiction)

   (c) * If G contains a $(w + 1)$-linked set of size at least $3w$, then G has tree-width at least w. (Check the script for a good approach to the proof)

2. **Tree Decompositions**

   Use the algorithm presented in the lecture to compute a tree decomposition of the graph below: