Prof. Dr. Alexander Bockmayr,
Prof. Dr. Knut Reinert,
Sandro Andreotti

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Discrete Mathematics for Bioinformatics (P1)

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Exercises 2

1. **Affine Combinations**
   a) Given $x^0, x^1, \ldots, x^k \in \mathbb{R}^n$ show that

$$\text{aff}(x^0, x^1, \ldots, x^k) = \{x^0\} + \text{lin}(x^1 - x^0, \ldots, x^k - x^0)$$

   b) $x^0, x^1, \ldots, x^k$ are called *affinely independent* if the unique solution of

$$\sum_{i=0}^{k} \lambda_i x^i = 0, \quad \sum_{i=0}^{k} \lambda_i = 0$$

   is $\lambda_i = 0$, for all $i = 0, \ldots, k$.
   How many affinely independent points exist in $\mathbb{R}^n$?

2. **Hulls**
   For the set of points

$$S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\} \subset \mathbb{R}^3$$

   give linear inequality descriptions of the polyhedra $\text{lin}(S), \text{cone}(S), \text{aff}(S), \text{conv}(S)$.

3. **Bases and Basic Solutions**
   Consider the polyhedron $P \subset \mathbb{R}^2$ defined by the system of linear inequalities

$$x_1 + x_2 \leq 2, \;\; 0 \leq x_1 \leq 1, \;\; 0 \leq x_2 \leq 2 \qquad (*)$$

   (a) Write (*) in the form $Ax \leq b$, for some $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.
   (b) Determine the bases of (*) and the corresponding basic solutions.
   (c) Which of them are feasible?
   (d) Give for each vertex of P the corresponding feasible bases.

4. **Simplex Algorithm**
   A farmer owns 100 acres of land. He can cultivate potatoes or corn.
   Given the data

	Potatoes	Corn	Available
Cultivation costs (in k€/a.)	1	2	110 k€
Working days per a.	1	4	160 working days
Profit (in k€/a.)	1	3	

   what should he plant in order to maximize his income ?

   (a) Model the problem as a linear optimization problem.

   (b) Solve it by applying the Simplex method.